Fermion localization on thick branes

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We consider chiral fermion confinement in scalar thick branes, which are known to localize gravity, coupled through a Yukawa term. The conditions for the confinement and their behavior in the thin-wall limit are found for various different BPS branes, including double walls and branes interpolating between different AdS_5 spacetimes. We show that only one massless chiral mode is localized in all these walls, whenever the wall thickness is keep finite. We also show that, independently of wall's thickness, chiral fermionic modes cannot be localized in dS_4 walls embedded in a M_5 spacetime. Finally, massive fermions in double wall spacetimes are also investigated. We find that, besides the massless chiral mode localization, these double walls support quasi-localized massive modes of both chiralities.

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I. INTRODUCTION

It is a well-known fact that fermions cannot be localized in Randall-Sundrum [1] branes. The fermion chiral modes turn out to be proportional to the inverse warp factor, so that the same effect that allows to confine gravitons forces the fermions out of the brane. This can be avoided if the brane is in fact a domain wall generated by the vacuum expectation value of a scalar field, as suggested long time ago by Rubakov and Shaposhnikov [2], and fermions are coupled to the scalar field. Bajc and Gabadadze [3] showed how this can be achieved by introducing a Yukawa coupling with the scalar field, assuming an infinitely thin kink profile for the wall.

However, in order to have a consistent kink profile, the scalar field should be a solution to the coupled Einstein-scalar field system with a suitable symmetry breaking potential and the thin brane must be obtained as the thin wall limit of this solution. Thin wall geometries have distribution-valued curvatures whose singular parts are proportional to a Dirac distribution supported on the surface where the wall is localized, and it is known that strong conditions must be imposed on a spacetime metric in order to ensure that its curvature tensors converge (in the sense of distributions) to the curvature tensors of the limit metric [4]. One of the relevant consequences of these restrictions is that domain wall solutions cannot be made infinitely thin while keeping the asymptotic values of the scalar field fixed, as in the step-function kink of [3].

Localization of gravity on thick domain walls has been considered in various works [5, 6, 7, 8, 9, 10, 11] and the thin wall limit (in the above mentioned distributional convergence sense) in [11, 12, 13]. Some of these thick branes reduce to the Randall-Sundrum (RS) brane [1] in the thin-wall limit, but not all of them, notably dynamic walls [12, 14], walls that interpolate between AdS_5 spacetimes with different cosmological constants [11] and double-walls [10, 13]. In some cases, as in the asymmetric branes found in [15], localization of gravity is not possible [11], although the scalar field behaves as a domain wall and the stress-energy tensor is distributionally well defined even for thickness zero.

It is therefore of interest to investigate the possibility that fermion confinement, being directly dependent on the scalar field solution and not only on the spacetime metric, can be affected by the internal structure on the thick brane. Confinement of fermions in general spacetimes has been addressed for example in [16]. Localization of chiral fermions in the regularized RS scenario of [6, 7, 8] in [7]. In [17], it is found that allowing for an 5-dimensional fermion mass term can result in the confinement of both left and right modes, although a marginal one. Namely, fermions can escape into the bulk by tunneling, and the rate depends on the parameters of the scalar field potential. Confinement of massive fermions on a numerical thick brane solution, has been considered in [18], introducing an effective 4-dimensional mass term on the brane. In [19], the same results are found using the analytical solution of [5].

In this paper we address the issue of chiral fermion mode confinement in thick branes, in particular those that exhibit an interesting internal structure. After reviewing this confinement in two BPS regularized versions of the RS brane [7, 19], we consider the asymmetric BPS branes [11], where the wall separates spacetimes with different cosmological constants, and find the conditions for fermion mode localization. The Yukawa coupling required to localize chiral fermions is shown to diverge in the thin-wall limit for all these BPS branes. We then turn to spacetimes with a dS_4 thick brane embedded in a M_5 bulk, whose four dimensional version was found in [14]. These are known to localize gravity [9, 22] and could be of cosmological interest. However, we find that these walls cannot localize fermion modes, independently of the wall's thickness. Finally, we explore the double walls found in [13]. We show that massless chiral fermions can be confined in these walls, and that in the massive case they can allow for left- and

right-handed mode quasi-confinement.

II. LOCALIZATION OF MASSLESS FERMIONS

Let us first consider a domain wall solution of the coupled Einstein-scalar fields equations, with an asymptotically AdS_5 metric of the form

$$ds^2 = e^{2A(r)}\eta_{\mu\nu}dx^\mu dx^\nu + dr^2 \tag{1}$$

where η is the 4-dimensional Minkowski metric (in the following a capital index like M runs from 0 to 4 while greek indices run from 0 to 3, we use units where $8\pi G_5 = 1$, with G_5 the gravitational constant in five dimensional spacetime).

The 5-dimensional spinor field satisfies the massless Dirac equation in the background (1)

$$\Gamma^M \nabla_M \Psi(x, r) = 0 \tag{2}$$

With the decomposition

$$\Psi(x, r)_\pm = \psi(x)_\pm f(r)_\pm \tag{3}$$

and demanding $\psi(x)_\pm$ satisfy the massless 4-dimensional Dirac equation, the equation for the chiral modes is integrated to give [3]

$$f(r)_\pm \propto e^{-2A(r)} \tag{4}$$

So that the very fact that the gravitational modes are confined to the wall, i.e. $e^{2A(r)} \to 0$ as $|r| \to \infty$, implies that $f(r)_\pm$ is not normalizable, i.e. that the fermionic fields cannot be confined. In order to have fermions on the wall, one can couple them to the scalar field with a Yukawa term [3] of the form $\lambda\overline{\Psi}\Psi\phi$, and then get

$$f(r)_\pm \propto e^{-2A(r) \pm \lambda \int \phi(r) dr} \tag{5}$$

For appropriate values of λ, the function $f(r)_-$ can in principle be normalized and one chiral fermion mode confined. Let us consider some specific solutions for the domain wall $\phi(r)$

A. The Kink

The solution

$$\phi = \phi_0 \tanh\left(\frac{\alpha r}{\delta}\right); \qquad \phi_0 \equiv \sqrt{3\delta} \tag{6}$$

for the Einstein-scalar field system with a symmetry breaking potential $V(\phi)$ is found when [6]

$$A(r) = -\frac{2}{3}\delta\left[\ln\left(\cosh\left(\frac{\alpha r}{\delta}\right)\right) + \frac{1}{4}\tanh^2\left(\frac{\alpha r}{\delta}\right)\right] \tag{7}$$

with

$$V(\phi) = \frac{9}{2}\alpha^2\left[\frac{1}{\phi_0^2}\left(1 - \frac{\phi^2}{\phi_0^2}\right)^2 - \frac{4}{3}\left(\frac{\phi}{\phi_0}\right)^2\left(1 - \frac{1}{3}\left(\frac{\phi}{\phi_0}\right)^2\right)^2\right] \tag{8}$$

and the AdS_5 cosmological constant is given by $\Lambda = -8\alpha^2/3$. Notice that although $V(\phi)$ as given by (8) is unbounded from below, a stability argument exists for this kind of BPS walls [20]. Confinement of gravitational modes for this model, in different parameterized forms, is discussed in [7, 8].

The thin-wall limit is found for $\delta \to 0$ and corresponds to $A(r) \to -2\alpha|r|/3$, which behaves as the usual warp factor of the RS geometry, while $\phi \to 0$ in this limit. Actually, following [12], it can be proved that (1,7) provides a sequence of metrics that satisfies the required convergence conditions of [4]. Then the distributional limit $\delta \to 0$ of

all the curvature tensor fields of (1,7) exists and gives the curvatures of the limit metric. This rather technical proof will not be presented here.

Now, for this domain wall, it is readily found using (5) that one chiral fermion mode is localized on the brane when

$$\lambda > \sqrt{\frac{|\Lambda|}{2\delta}} \tag{9}$$

This result, in a slightly different parameterized form, has been found in [7]. It should be noted that, strictly speaking, there is no confinement in the infinitely-thin wall.

B. Domain wall for a sine-Gordon potential

A solution with

$$\phi = \sqrt{3\delta}\,\arctan\left(\sinh\left(\frac{\alpha r}{\delta}\right)\right) \tag{10}$$

can be found [5] for the metric (1) with

$$A(r) = -\delta\ln\left(\cosh\left(\frac{\alpha r}{\delta}\right)\right) \tag{11}$$

and the scalar field potential

$$V = 3\alpha^2\left[\left(\frac{1}{2\delta}+2\right)\cos^2\left(\frac{\phi}{\sqrt{3\delta}}\right)-2\right] \tag{12}$$

The parameter α is given by the cosmological constant, $\Lambda = -6\alpha^2$. This smooth domain wall geometry (parameterized in a sligtly different form) has been shown to localize four-dimensional gravity on the wall [5].

The thin-wall limit for this solution is found for $\delta \to 0$ and corresponds to $A(r) \to -\alpha|r|$, the usual warp factor of the RS geometry. It can be shown that in this limit the energy-momentum tensor and all the curvatures converge rigourously to the corresponding tensor distributions associated to the RS thin brane geometry [12, 21]), with singular parts which are proportional to a δ-distribution supported on the wall's plane. Now, it should be noticed that the scalar field (10) vanishes everywhere in the thin-wall limit $\delta \to 0$. Since δ is the only parameter that can be consistently interpreted as the wall's thickness, it is not possible to take the thin-wall limit rigorously while keeping $\phi \neq 0$.

Now, equation (5) gives

$$f(r)_\pm \propto \left(\cosh\left(\frac{\alpha r}{\delta}\right)\right)^{2\delta}\exp\left\{\pm\lambda\sqrt{3}\,\frac{\delta^{3/2}}{\alpha}\sum_{k=0}^{\infty}\frac{E_k\left[\arctan(\sinh\alpha r/\delta)\right]^{2k+2}}{(2k+2)(2k)!}\right\} \tag{13}$$

where E_k are the Euler's numbers ($E_1 = 1$, $E_2 = 5$, $E_3 = 61$, etc. and $E_0 = 1$), which can be rewritten as

$$f(r)_\pm \propto \exp\left\{\sum_{k=1}^{\infty}\left[\frac{\delta\,2^{2k}\,(2^k-1)\,B_k}{k\,(2k)!} \pm \frac{\lambda\sqrt{3}\,\delta^{3/2}\,E_{k-1}}{2\,\alpha\,k\,(2k-2)!}\right]\left[\arctan(\sinh\alpha r/\delta)\right]^{2k}\right\} \tag{14}$$

where B_k are the Bernoulli's numbers ($B_1 = 1/6$, $B_2 = 1/30$, $B_3 = 1/42$, etc.) From (14) it follows that in order to confine one of the chiral modes, one must require

$$\lambda > \frac{2\alpha}{3\sqrt{3\delta}} = \frac{1}{3^2}\sqrt{\frac{2|\Lambda|}{\delta}} \tag{15}$$

Clearly in this model, as in the previous one, there is no confinement in the infinitely-thin wall.

Localization of fermions on this thick brane, with a different parametrization and employing the asymptotic values of ϕ as $r \to \pm\infty$ [3] in (5) instead of evaluating explicitly the integral as we have done, is discussed in [19]. Here it is instructive to compare the result obtained in this way with the exact result (15). In the large r limit, we find that asymptotically

$$f(r)_\pm \propto \exp\left(2\alpha \pm \lambda\sqrt{3\delta}\,\frac{\pi}{2}\right)|r| \tag{16}$$

so that in order to confine one of the chiral modes, one must have

$$\lambda > \frac{4\alpha}{\pi\sqrt{3\delta}} = \frac{2}{3\pi}\sqrt{\frac{2|\Lambda|}{\delta}} \tag{17}$$

Notice that solutions to the Einstein-scalar field equations with a plane-parallel symmetry with a given $V(\phi)$ and $\phi(r)$ are always part of a family of "shifted" solutions with $\tilde{\phi}(r) = \phi(r) - \epsilon$ and $\tilde{V}(\tilde{\phi}) = V(\tilde{\phi} + \epsilon)$, with ϵ a constant, having the same metric and cosmological constant. The shifting is irrelevant for the gravitational properties, but not for the fermion confinement, as is easily seen by setting $\epsilon \to \phi(r = \infty)$: fermion modes can then escape to infinity, or in other words, the Yukawa coupling required is infinite. In the case of wall separating spacetimes with the same cosmological constant, the Yukawa coupling required is minimal for $\epsilon = 0$, the symmetric kink solution. But this is not the case for asymmetric walls, as we consider now.

C. Asymmetric walls

An example of an asymmetric domain wall was found in [11], with

$$\phi = 2\sqrt{3\delta}\left[\exp\{-e^{-\beta r/\delta}\} - \epsilon\right] \tag{18}$$

$$V(\phi) = 18\left\{\left[\frac{\beta}{12\delta}\tilde{\phi}\ln\left(\frac{\tilde{\phi}^2}{12\delta}\right)\right]^2 - \left[\frac{\beta}{12\delta}\tilde{\phi}^2\left(1 - \ln\left(\frac{\tilde{\phi}^2}{12\delta}\right)\right) - \alpha\right]^2\right\}, \qquad \tilde{\phi} = \phi + 2\sqrt{3\delta}\epsilon \tag{19}$$

and a warp factor

$$A(r) = \alpha r - \delta\exp(-2e^{-\beta r/\delta}) + \delta\,\mathrm{Ei}\left(-2e^{-\beta r/\delta}\right), \tag{20}$$

where the exponential integral is given by $\mathrm{Ei}(u) \equiv -\int_{-u}^{\infty} e^{-\tau}/\tau\, d\tau$ (see [11] for details). This is a family of domain wall spacetimes without reflection symmetry along the direction perpendicular to the wall. Far from the wall, the warp factor is

$$A(r \to -\infty) = \alpha\, r, \qquad A(r \to \infty) = -(\beta - \alpha)\, r \tag{21}$$

The wall thus interpolates between AdS_5 spacetimes with cosmological constant $\Lambda_- = -6\alpha^2$ for $r < 0$ and $\Lambda_+ = -6(\beta - \alpha)^2$ for $r > 0$. The constant ϵ fixes the asymptotic values of the scalar field at $r = \pm\infty$ and δ is the wall's thickness. This asymmetric domain wall geometry (for $\beta > \alpha > 0$) has been shown to localize four-dimensional gravity on the wall [11].

Now, equation (5) gives

$$f(r)_\pm \propto -2\left[\alpha r - \delta\exp(-2e^{-\beta r/\delta}) + \delta\,\mathrm{Ei}\left(-2e^{-\beta r/\delta}\right)\right] \pm 2\,\lambda\,\sqrt{3\delta}\left[-\frac{\delta}{\beta}\,\mathrm{Ei}\left(-e^{-\beta r/\delta}\right) - \epsilon\, r\right] \tag{22}$$

whose behavior is however rather complicated. Again, one can study the asymptotic behavior of the chiral modes. We have for the ϕ field

$$\phi(r \to -\infty) = -2\sqrt{3\delta}\epsilon \qquad \phi(r \to \infty) = -2\sqrt{3\delta}(1 - \epsilon) \tag{23}$$

The Yukawa coupling required to confine fermions to these walls is minimized when

$$\epsilon^{-1} = 1 + \sqrt{|\Lambda_+|/|\Lambda_-|} \tag{24}$$

and the condition for this confinement reads

$$\lambda > \frac{1}{3\sqrt{2\delta}}(\sqrt{|\Lambda_+|} + \sqrt{|\Lambda_-|}) \tag{25}$$

Since the scalar field depends on $\beta \propto \sqrt{|\Lambda_+|} + \sqrt{|\Lambda_-|}$, fermions are confined to the wall as long as one of the cosmological constants is non-zero; of course gravity is not localized if $\Lambda_+ = 0$ or $\Lambda_- = 0$.

D. A de Sitter thick brane

A domain wall solution interpolating between spacetimes without a 5-dimensional cosmological constant can be found if one allows for a de Sitter expansion on the wall plane, i.e. a dynamic solution [14], that can be of cosmological interest. The Einstein-scalar field system can be integrated for a metric

$$ds^2 = e^{2A(\bar{r})}(-dt^2 + e^{2\beta t}\, dx^i\, dx^i + d\bar{r}^2) \tag{26}$$

where $i = 1, 2, 3$.

The scalar field is given by

$$\phi(\bar{r}) = \sqrt{3\delta(1-\delta)}\,\arctan\left(\sinh\left(\frac{\beta\bar{r}}{\delta}\right)\right) \tag{27}$$

with

$$A(\bar{r}) = -\delta\ln\left(\cosh\left(\frac{\beta\bar{r}}{\delta}\right)\right) \tag{28}$$

and a sine-Gordon scalar field potential. These walls have a well-defined distributional thin limit $\delta \to 0$, a thin dS_4 brane embedded in a M_5 bulk [12, 21]). Although they do not become an RS brane in the thin wall limit, they can localize gravity with a mass gap between the zero modes and the massive ones [9, 11, 22]. However, they cannot confine fermions as we now show.

In these coordinates, the Dirac equation results in a different expression for $f_\pm$

$$f_\pm \propto \exp\left\{-2A(\bar{r}) \pm \lambda \int e^{A(\bar{r})}\phi(\bar{r})d\bar{r}\right\} \tag{29}$$

which can be explicitly integrated and yields a power series of $\arctan(\sinh\alpha r/\delta)$ as in (14), but quite more involved with coefficients which are polynomials in δ. Each term of the series, for increasing power, pushes up the lower bound on λ necessary to obtain the desired normalizability of $f_\pm$. Hence, there is no value of λ that renders $f_\pm$ normalizable. This is not at all unexpected, in this model $A(\bar{r})$ is the same function $A(r)$ of the domain wall with a sine-Gordon potential and $f_\pm$ can be rewritten as

$$f_\pm \propto \exp\int d\bar{r}\,\Upsilon(\bar{r})_\pm \tag{30}$$

where

$$\Upsilon(\bar{r})_\pm = \left[2\,\alpha\,\tanh\frac{\alpha\bar{r}}{\delta} \pm \lambda\,\sqrt{3\delta(1-\delta)}\,\exp(-\delta\ln(\cosh\frac{\alpha\bar{r}}{\delta}))\,\arctan(\sinh\frac{\alpha\bar{r}}{\delta})\right] \tag{31}$$

It follows that the Yukawa coupling contributes to $\Upsilon(\bar{r})_\pm$ with a term that becomes exponentially suppressed with respect to the one of the domain wall of the sine-Gordon potential, becoming unable to cancel the coming one from $\exp\{-2\,A(\bar{r})\}$ for $|\bar{r}| \to \infty$. Asymptotically,

$$\Upsilon(\bar{r})_- \to \pm\left(2\alpha - \lambda\sqrt{3\delta(1-\delta)}\,e^{-\beta|\bar{r}|}\,\frac{\pi}{2}\right) \tag{32}$$

as $\bar{r} \to \pm\infty$ and the term coming from the Yukawa coupling goes to zero. This behavior has also been checked by numerical integration of (29).

E. Double walls

In [13], a two-parameter family of walls are found with a metric

$$ds^2 = e^{2A(\bar{r})}\left(\eta_{\mu\nu}dx^\mu dx^\nu + d\bar{r}^2\right) \tag{33}$$

The scalar field solution is

$$\phi(\bar{r}) = \phi_0\arctan(\alpha\bar{r})^s\,;\ \phi_0 = \frac{\sqrt{6s-3}}{s} \tag{34}$$

with

$$A(\bar{r}) = -\frac{1}{2s}\ln\left(1+(\alpha\bar{r})^{2s}\right) \tag{35}$$

and

$$V = 3\alpha^2\sin(\phi/\phi_0)^{2-2/s}\left[\frac{2s+3}{2}\cos^2(\phi/\phi_0)-2\right] \tag{36}$$

If s is an odd integer, the field (34) interpolates between two global minima of the potential, but for $s > 1$ there is a local minima between any two global ones. The configuration then is that of a double wall, with the energy density peaking at two points, interpolating between AdS_5 asymptotic vacua with $\Lambda = -6\alpha^2$. These walls have been shown to confine gravity in [11]; similar double walls have been found in [10]. For $s = 1$, this is just the RS regularized spacetime considered in II B, for a fixed thickness $\delta = 1$ and written in conformal coordinates.

The chiral modes take again the form (29), but in contrast with the dynamic walls, confinement of chiral fermions is possible in this case, for any odd value of s. The integration of (29) in this case has again been performed numerically, giving a renormalizable mode. But it is also possible in this case to examine the asymptotic behavior of the integrand, writing the modes as in (30) we find

$$\Upsilon(\bar{r})_{\pm} \to \left(\frac{2}{\bar{r}} \pm \lambda\frac{\pi}{2}\frac{\sqrt{6s-3}}{s}\frac{1}{\alpha\bar{r}}\right) \tag{37}$$

as $|\bar{r}| \to \infty$. So that the Yukawa term dominates when

$$\lambda > \frac{4\alpha}{\pi\sqrt{3}}\frac{s}{\sqrt{2s-1}} \tag{38}$$

III. LOCALIZATION OF MASSIVE FERMIONS

Up to now, we have supposed that the 4-dimensional spinor fields satisfy the massless Dirac equation on the wall. The inclusion of a mass term that connects left and right-handed modes could modify the well-known result of having only one chiral mode localized on the brane. This possibility has been explored in Ref. [18], using a numerical solution for the domain wall. If instead of this one uses the exact solutions (10,11) as in Ref. [19], or (6,7), very similar results are found. Namely, the chiral modes can be shown to satisfy a Schrödinger-like equation, with different potentials for left and right-modes. These potentials are unbounded from below, but have minima where the modes can be confined, and the tunneling can in principle be sufficiently suppressed by adjusting the parameters.

Here we study the case of the double wall spacetime of Ref. [13], which includes the regularized version of the RS geometry considered in [5] as a particular case. These branes have the advantage of being written in a coordinate system that leads to a Schrödinger equation for the chiral modes with the four-dimensional fermion masses as its eigenvalues, which enable us to analyze its solutions in a simpler way.

Consider again a spinor Ψ in 5 dimensions,

$$\Psi(x,\bar{r}) = \psi_+(x)u_+(\bar{r}) + \psi_-(x)u_-(\bar{r}) \tag{39}$$

where now the 4-dimensional spinors satisfy

$$i\gamma^{\mu}\nabla_{\mu}\psi_{\pm} = m\psi_{\mp} \tag{40}$$

Then the chiral modes follow the equations (prime denotes derivative respect to $\bar{r}$)

$$\left[\partial_r + 2A'(\bar{r}) \pm \lambda\phi(\bar{r})e^{A(\bar{r})}\right]u_{\mp} = \pm\, m\, u_{\pm} \tag{41}$$

Setting

$$u(\bar{r}) = \hat{u}(\bar{r})e^{-2A(\bar{r})} \tag{42}$$

one can write the Schrödinger equations

$$\left[-\partial_r^2 + V_{QM}^{\pm}\right]\hat{u}_{\pm} = m^2\hat{u}_{\pm} \tag{43}$$

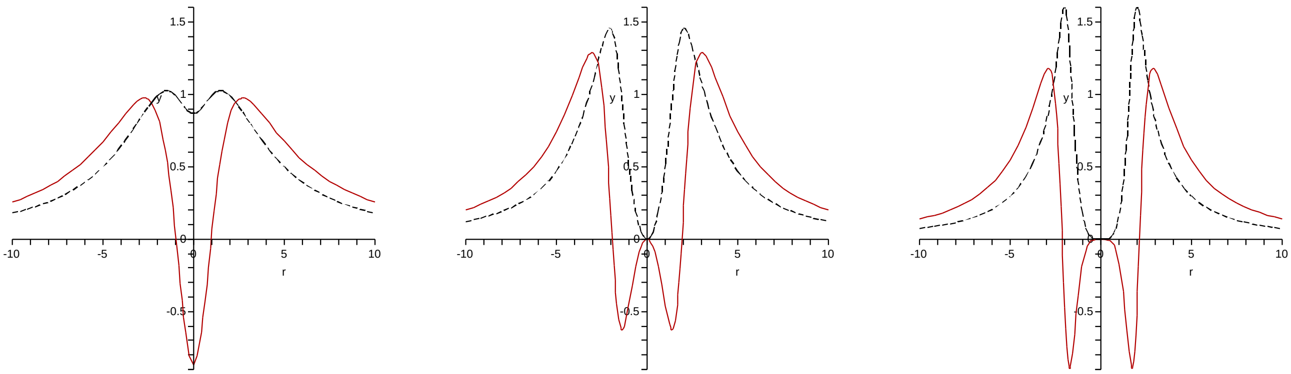

FIG. 1: V_{QM}^{-} (continuous line) and V_{QM}^{+} (dashed line) for the double walls with $s = 1$, $s = 3$ and $s = 5$

where

$$V_{QM}^{\pm} = (\lambda\phi(\bar{r})e^{A(\bar{r})})^2 \pm \partial_r(\lambda\phi(\bar{r})e^{A(\bar{r})}) \tag{44}$$

In figure 1 we plot $V_{QM}^{\pm}$ for different values of s. In all cases $\alpha = 1$, $\lambda = 1$.

Now, notice that (43) can be written as

$$Q^{+}Q\,\hat{u}_{-} = m^2\,\hat{u}_{-}, \qquad Q\,Q^{+}\,\hat{u}_{+} = m^2\,\hat{u}_{+} \tag{45}$$

where $Q \equiv \partial_{\bar{r}} + \lambda\,\phi(\bar{r})\,\exp A(\bar{r})$ and $Q^{+} \equiv -\partial_{\bar{r}} + \lambda\,\phi(\bar{r})\,\exp A(\bar{r})$, and (43) can be recast as a SUSY quantum mechanics problem as in [23]. It follows that the eigenvalues of the $\hat{u}_{-}$ and $\hat{u}_{+}$ modes always come in pairs, except possibly for the massless modes. Indeed, this pairing of the mass eigenmodes is required for the existence of massive fermions satisfying (40).

As a simple check of consistency, let us consider the massless modes again. For $m = 0$, from (45) we find

$$\hat{u}_{0\pm} \propto \exp\{\pm\lambda \int d\bar{r}\,\phi\,e^{A}\} \tag{46}$$

and from (42) the result (29) is recovered. Since for

$$\lambda > \frac{4\alpha}{\pi\sqrt{3}}\frac{s}{\sqrt{2s-1}} \tag{47}$$

one and only one chiral mode is normalizable, this chiral normalizable mode is just the one that was considered in the previous section and that gives a chiral four dimensional fermion localized on the brane.

Now let us consider the massive modes. We recall that for $s = 1$, this is just the second RS regularized spacetime considered in the previous section, for a fixed thickness $\delta = 1$ and written in conformal coordinates. For $s = 1$, we see from Fig. 1 that right- and left-handed modes are trapped in a metastable state with a non-zero probability of tunneling into the bulk. This tunneling probability depends on the value of the Yukawa coupling and could be made small by increasing λ. For the range of parameters shown, this tunneling probability is very high, to the point that one can exclude localization. However, for the same set of parameters but greater values of s, where double walls appear, we see from Fig. 1 that V_{QM}^{+} develops a deep minimum in the region between the walls. Now, right-handed modes of small mass can be trapped here in a metastable state like the paired massive left-handed ones. As before, the tunneling probability can be made small by increasing λ. On the other hand, increasing s has the effect of making each of the walls thinner, accordingly, V_{QM}^{+} has also thinner "walls", which would increase the probability of fermions escaping. A precise calculation of the tunneling probability for these massive left and right-handed modes is in order here, but will only make sense in a definite model in which the parameters of the potential can be related with the age of the Universe. This is beyond the scope of this paper and will be dealt with elsewhere [24]. It is however clear that the internal structure of the wall, i.e. the appeareance of double walls, can enhance the probability of localizing massive fermions.

IV. SUMMARY

We have studied the localization of fermions on thick branes with a non-trivial internal structure. We find that double walls can confine fermions of a given chirality, as is also the case with the known regularized RS branes, provided they are coupled to the scalar field through a Yukawa coupling that is inversely proportional to the wall's thickness. Asymmetric BPS walls interpolating between spacetimes with different cosmological constants can confine fermions in similar way, with a Yukawa coupling proportional to the sum of the square roots of the cosmological constants. In all these cases, the Yukawa coupling required diverges in the thin-wall limit. On the other hand, dynamic walls with a cosmological de Sitter expansion are unable to localize even one chiral mode. We have considered here only the five dimensional version of the well-known solution of Goetz [14], but these results are expected to hold for other dynamic walls such as the ones found recently in [25]; work in this direction is now in progress [24].

Finally, we have also addressed the issue of confinement of massive fermions in double walls. We find that, besides the massless chiral mode localization, massive modes of both chiralities can be quasi-localized on double walls, being "trapped" in between the walls. The tunneling probability can in principle be made as small as desired by increasing the Yukawa coupling and fermion masses, but for a given set of parameters it is smaller in the double walls than in the single ones. Thus, fermion localization is shown to be affected by the internal structure of the domain wall solutions.

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[1] L. Randall and R. Sundrum, Phys. Rev. Lett. **83** (1999) 4690 [arXiv:hep-th/9906064]. See also M. Gogberashvili, Int. J. Mod. Phys. D **11** (2002) 1635 [arXiv:hep-ph/9812296].
[2] V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B **125** (1983) 136.
[3] B. Bajc and G. Gabadadze, Phys. Lett. B **474** (2000) 282 [arXiv:hep-th/9912232].
[4] R. Geroch and J. H. Traschen, Phys. Rev. D **36** (1987) 1017.
[5] M. Gremm, Phys. Lett. B **478** (2000) 434 [arXiv:hep-th/9912060].
[6] O. DeWolfe, D. Z. Freedman, S. S. Gubser and A. Karch, Phys. Rev. D **62** (2000) 046008 [arXiv:hep-th/9909134].
[7] A. Kehagias and K. Tamvakis, Phys. Lett. B **504** (2001) 38 [arXiv:hep-th/0010112].
[8] Z. Kakushadze and P. Langfelder, Mod. Phys. Lett. A **15** (2000) 2265 [arXiv:hep-th/0011245].
[9] A. z. Wang, Phys. Rev. D **66** (2002) 024024 [arXiv:hep-th/0201051].
[10] D. Bazeia, C. Furtado and A. R. Gomes, JCAP **0402** (2004) 002 [arXiv:hep-th/0308034]. See also D. Bazeia, J. Menezes and R. Menezes, Phys. Rev. Lett. **91** (2003) 241601 [arXiv:hep-th/0305234].
[11] O. Castillo-Felisola, A. Melfo, N. Pantoja and A. Ramirez, Phys. Rev. D **70** (2004) 104029 [arXiv:hep-th/0404083].
[12] R. Guerrero, A. Melfo and N. Pantoja, Phys. Rev. D **65** (2002) 125010 [arXiv:gr-qc/0202011].
[13] A. Melfo, N. Pantoja and A. Skirzewski, Phys. Rev. D **67** (2003) 105003 [arXiv:gr-qc/0211081].
[14] G. Goetz, J. Math. Phys. **31** (1990) 2683.
[15] R. Gass and M. Mukherjee, Phys. Rev. D **60** (1999) 065011 [arXiv:gr-qc/9903012].
[16] S. Randjbar-Daemi and M. E. Shaposhnikov, Phys. Lett. B **492** (2000) 361 [arXiv:hep-th/0008079].
[17] S. L. Dubovsky, V. A. Rubakov and P. G. Tinyakov, Phys. Rev. D **62** (2000) 105011 [arXiv:hep-th/0006046].
[18] C. Ringeval, P. Peter and J. P. Uzan, Phys. Rev. D **65** (2002) 044016 [arXiv:hep-th/0109194].
[19] R. Koley and S. Kar, Class. Quant. Grav. **22** (2005) 753 [arXiv:hep-th/0407158].
[20] K. Skenderis and P. K. Townsend, Phys. Lett. B **468** (1999) 46 [arXiv:hep-th/9909070].
[21] N. Pantoja and A. Sanoja, J. Math. Phys. **46** (2005) 033509-1-13 [arXiv:gr-qc/0312032].
[22] A. Kehagias and K. Tamvakis, Class. Quant. Grav. **19** (2002) L185 [arXiv:hep-th/0205009].
[23] N. Arkani-Hamed and M. Schmaltz, Phys. Rev. D **61**, 033005 (2000) [arXiv:hep-ph/9903417].
[24] R. Guerrero, A. Melfo, N. Pantoja and R.O. Rodríguez, work in preparation.
[25] R. Guerrero, R. O. Rodriguez and R. Torrealba, arXiv:hep-th/0510023;
R. Guerrero, R. Ortiz, R. O. Rodriguez and R. Torrealba, arXiv:gr-qc/0504080.